ELLIPSOIDAL CONCENTRATORS FOR LABORATORY X-RAY SOURCES: ANALYTICAL OPTIMIZATION

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The conventional approach for optimization of the ellipsoidal monocapillary concentrator parameters consists in numerical simulation, e.g., with the use of ray-tracing technique. In the present work the optimization problem is solved analytically.

1. Point source

It is easy to show that efficiency of the ellipsoidal concentrator (the power transmission coefficient) for a point source can be expressed:

$$\nu_W \equiv \frac{W}{W_0} = \int_{\varphi_{min}}^{\varphi_{max}} R(\theta)\sin(2\varphi)\,d\varphi, \qquad (1)$$

W_0 – the power radiated by the point source to the concentrator half-space, W – the power concentrated in the ellipsoid focus, φ – angle between the ray and the symmetrical axis of the ellipsoid, φ_{max} and φ_{min} are defined by the positions of the front and back ends of the concentrator, θ – grazing angle, $R(\theta)$ – Fresnel reflection coefficient depending on θ and the dielectric permittivity of a reflecting coating ε ($\varepsilon = 1 - \delta + i\gamma$).

With approximations we obtain:

$$\nu_W = \frac{W}{W_0} = 4\sqrt{\delta^2+\gamma^2}\cdot Y(\tau_{min}, \gamma/\delta), \quad Y \approx \frac{1}{2}\int_{\eta_{min}}^{\eta_{max}} R\left(\tau, \frac{\gamma}{\delta}\right)\eta\, d\eta; \quad R\left(\tau,\frac{\gamma}{\delta}\right) = \left|\tau - \sqrt{\tau^2 - \exp(i\alpha)}\right|^4;$$

$$\tau \equiv \frac{\theta}{|1-\varepsilon|^{1/2}}, \quad \eta \equiv \frac{\varphi}{|1-\varepsilon|^{1/2}}, \quad \tau = \frac{\eta}{2} + \frac{\tau_{min}^2}{2\eta}; \quad \tau_{min} \equiv \left[\frac{2(1-e)}{|1-\varepsilon|}\right]^{1/2}; \quad \exp(i\alpha) \equiv \frac{1 - i\gamma/\delta}{\sqrt{1+\gamma^2/\delta^2}}. \qquad (2)$$

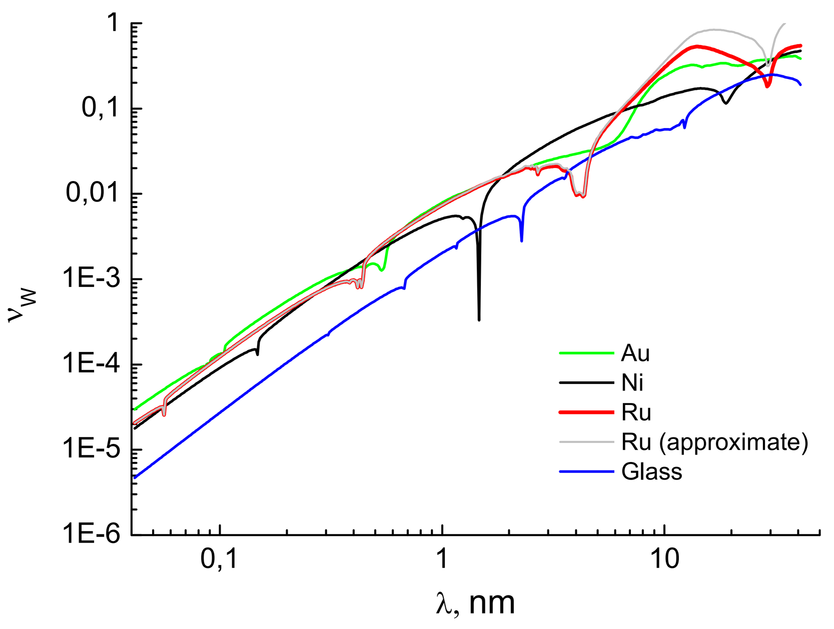


Figure 1. Parabolic concentrator efficiency dependence on the radiation wavelength for different reflecting materials. Approximate curve calculated by formula (2).

The Y value dependencies on the one of the approximate problem parameters γ/δ, τ_{min}, as well as $L/2F$ and η_{max} for the case of finite ellipsoid length L, are shown on fig. 2 – 5.

Figures 2 and 3 shows that necessary conditions for maximum efficiency are: <u>γ/δ **< 0.1**</u> and <u>τ_{min} **< 0.4**</u>.

In case of finite ellipsoid length the η integration limits are connected:

$$\eta_{max}^2 = \frac{\eta_{min}^2\left(1+\frac{L}{2F}\right) + \tau_{min}^2\frac{L}{2F}}{1 - \frac{L}{2F}\left(1+\frac{\eta_{min}^2}{\tau_{min}^2}\right)}$$

Thus Y is a function only of η_{max} for the finite length concentrator, if other parameters (γ/δ, τ_{min}, $L/2F$) are fixed.

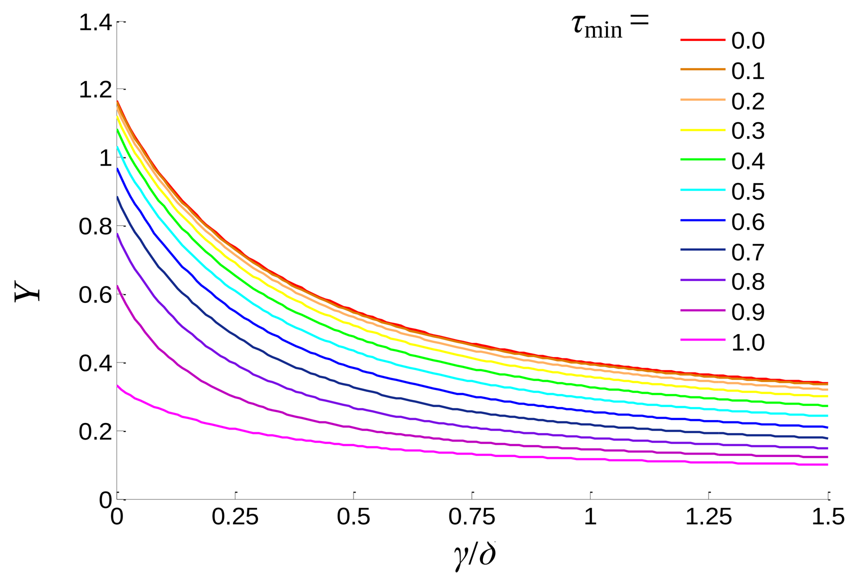


Figure 2. $Y(\gamma/\delta)$ for different τ_{min}.

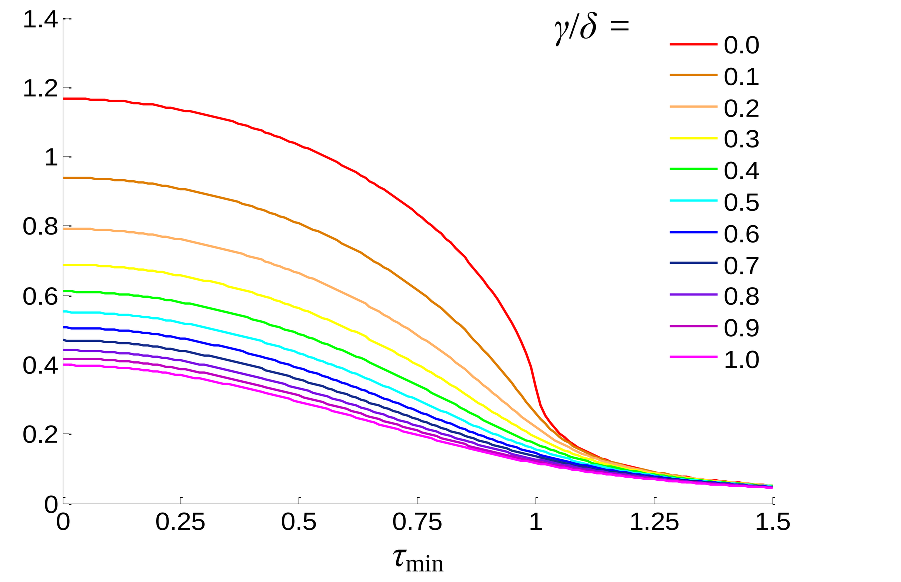


Figure 3. $Y(\tau_{min})$ for different γ/δ.

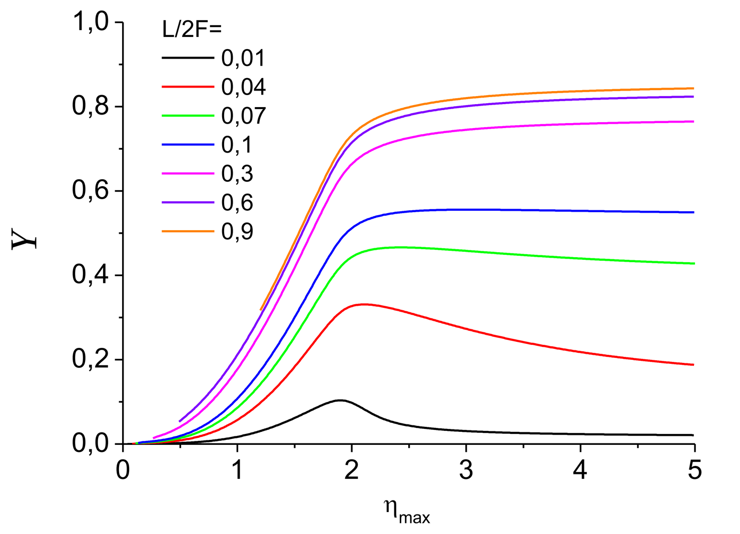


Figure 4. $Y(\eta_{max})$ for fixed $\gamma/\delta = 0.1$ и $\tau_{min} = 0.4$ and different values of $L/2F$.

Therefore, the optimization problem is reduced to finding the η_{max} value providing maximum of Y for given γ/δ, τ_{min} ans $L/2F$.

We see (fig. 4) that the local maximum disappears at $L/2F > 0.1$ and the maximum value of Y is reached at the maximum possible η_{max}. However, let's note that Y slightly increases when <u>η_{max} **> 3**</u>.

Figure 5 shows that the power transfer coefficient of the concentrator takes almost the maximum value when <u>**$L/2F$ > 0.5**</u>. A further increase of the concentrator length slightly raises its efficiency.

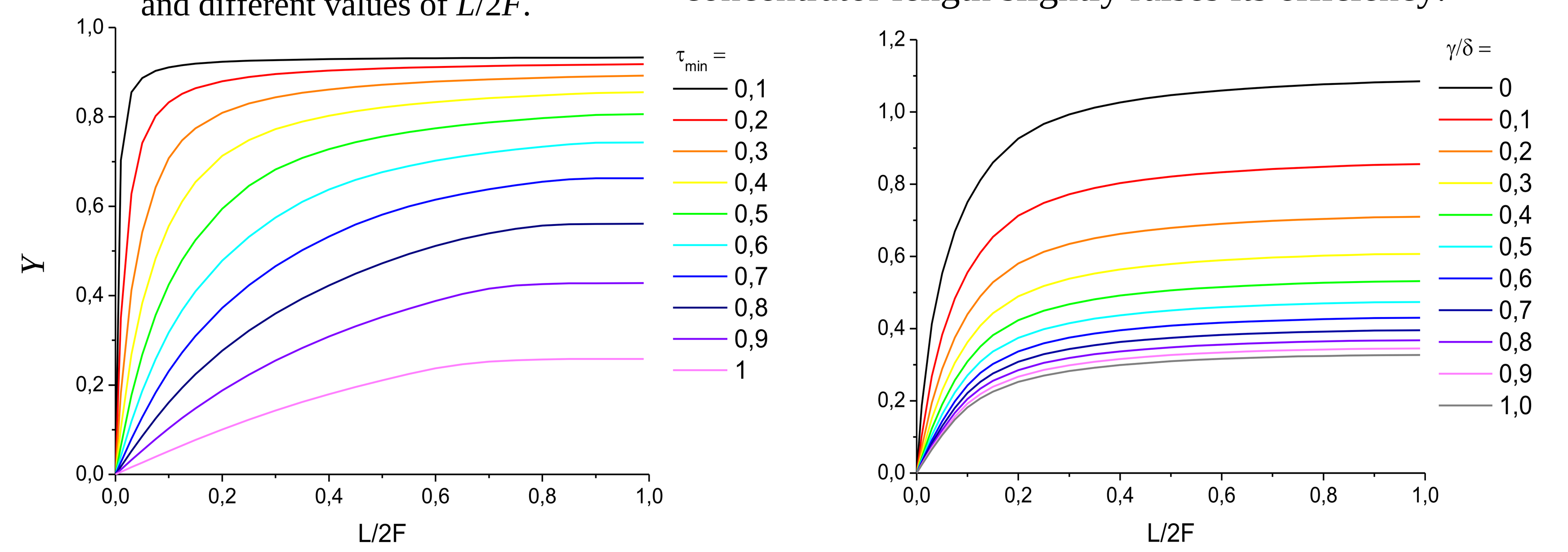
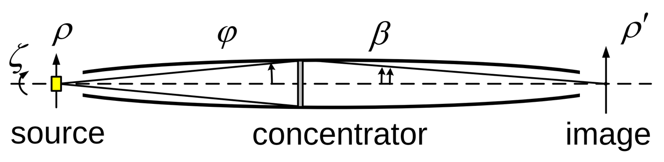


Figure 5. $Y(L/2F)$ for different τ_{min} and $\gamma/\delta = 0.1$ (left); for different γ/δ and $\tau_{min} = 0.4$ (right).

2. Source of the finite size

It is easy to show that the source of small size ρ transferred by the cross-section of concentrator to the spot in the focal plane with size:



$$\rho'(\varphi, \rho) = \rho\frac{\sin 2\varphi}{\sin 2\beta} = \rho \cdot M(\varphi)$$

Assuming that the grazing angle is the same for all points of a small source, and integrating over the azimuthal angle, we obtain an expression for the intensity distribution in the image plane:

$$I(\rho') = \int_{\varphi_{min}}^{\tilde{\varphi}_{max}(\rho')} d\varphi \cdot \frac{\pi b R(\theta(\varphi))\sin 2\varphi}{M(\varphi)^2}$$

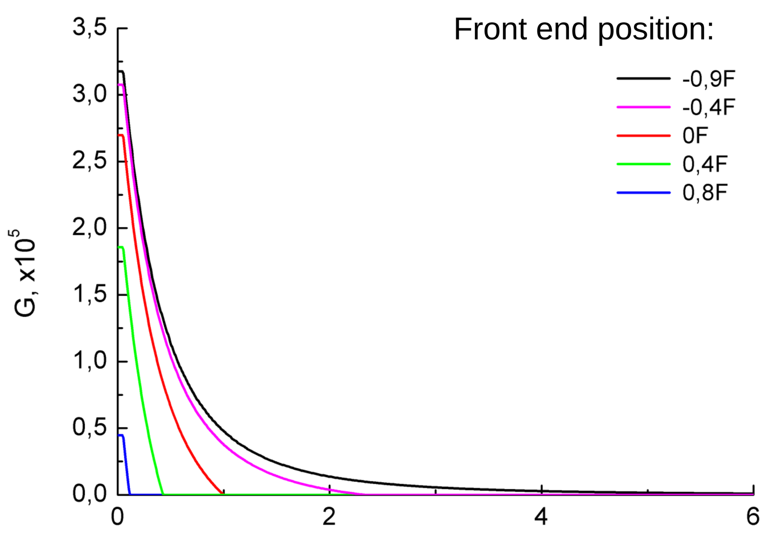


Figure 6. Normalized intensity distribution in the focal plane

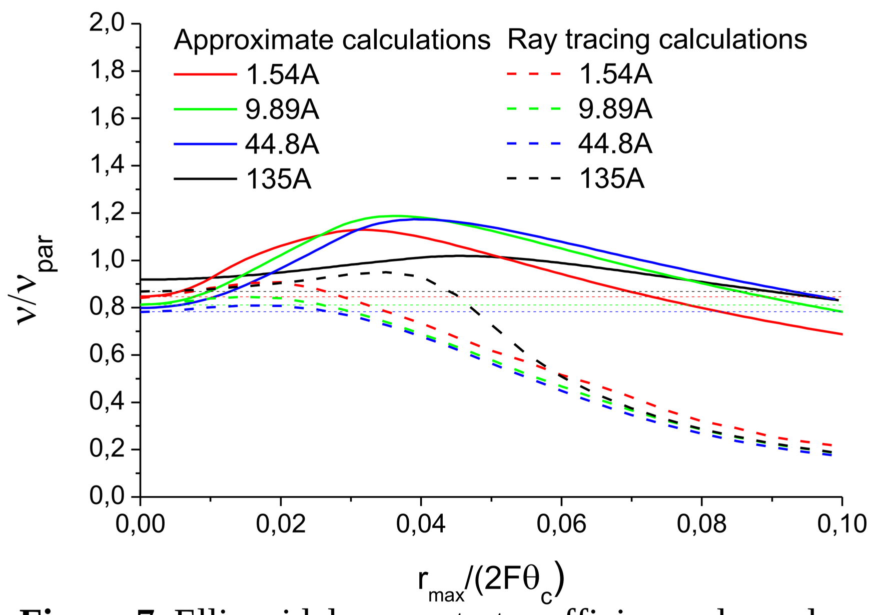


Figure 7. Ellipsoidal concentrator efficiency dependence on the source size for different wavelengths and reflecting coatings.

Figure 6 shows the results of calculations for the source size of 50 μm, glass ellipsoid with semi axes of 100 cm and 1 cm, fixed position of the back end (0.9F) and various positions of the front end. The resulting distributions were normalized to the total radiated power transmitted without reflection through the capillary.

However, this approach does not allow to study the influence of source size on the power transfer efficiency of the ellipsoid. Analytical estimates show that the power transmission coefficient varies slightly, while next condition is satisfied:

$$\frac{r_{max}}{2F\theta_c} < 0.5\cdot\left(1-\sqrt{1-\tau_{min}^2}\right), \quad \theta_c = |1-\varepsilon|^{1/2} \text{ – critical angle of total external reflection}$$

In order to find the power transmission coefficient it is necessary to calculate the following integral (in approximation of beam single reflection from the walls of the concentrator):

$$\nu = \frac{1}{S}\int_0^{r_{max}} r\,dr \int_0^{2\pi} d\zeta \int_{\varphi_{min}(\zeta,\vec{r})}^{\varphi_{max}(\zeta,\vec{r})} R\left(\theta(\varphi) - \frac{r}{2F}\left[1+\frac{\varphi^2}{2(1-e)}\right]\cos\zeta\right)\sin 2\varphi\, d\varphi$$

In the calculations (fig. 7) we assumed: $L/2F = 0.5$, $\tau_{min} = 0.4$. The results approve the estimates:

$$0.5\left(1-\sqrt{1-\tau_{min}^2}\right) \approx 0.042$$

We see that the power transfer efficiency even increases if this condition is satisfied, reaching a maximum in the vicinity of the source critical size r_{max}. Such unusual behavior of the concentrator efficiency may be due to the neglecting of multiplicity of ray reflections.

We decided to check this result by ray tracing technique. This simulation has approved increasing of power transmission coefficient with source size increasing till critical value (fig. 7).

Let's consider reflection of disk source radiation from one point on a surface to explain it (fig. 8). It's easy to find the curve corresponding to the source points which illuminate the reflection point at a fixed grazing angle (fig. 9).

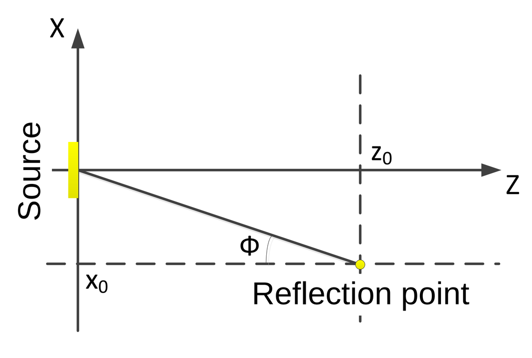


Figure 8. Reflection from surface point

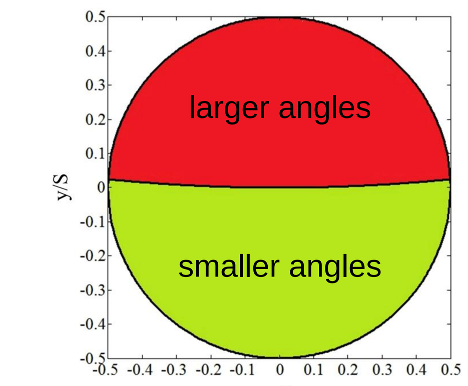


Figure 9. The source is divided on two parts: with less and greater grazing angle

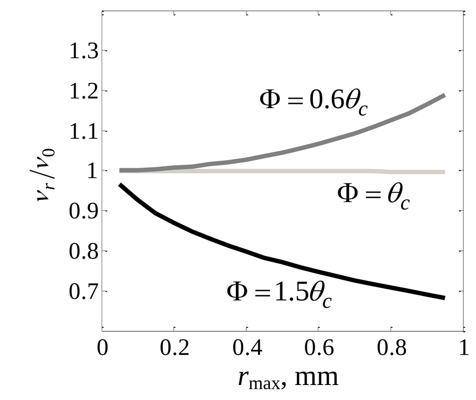


Figure 10. The power transmission coefficient dependence on the source size for different grazing angles Φ of point source

This curve divides the source into two unequal parts, bigger part corresponds to lower grazing angles. That is the possible reason for increasing of the concentrator efficiency with increasing of the source size (see Fig. 10).

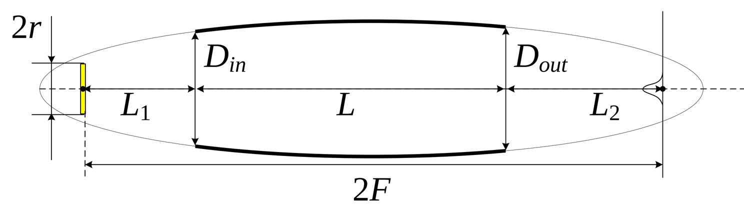


Figure 11. Ellipsoid parameters.

3. Conclusions

The optimization of ellipsoidal concentrators was solved analytically in the present work. The concentrator efficiency was expressed via several dimensionless parameters that allows us:

1) fast evaluation of the concentrator efficiency in the given conditions,

2) fast determination of the optimal concentrator parameters for given case (see Table).

We can assert that satisfaction of several conditions (<u>γ/δ **< 0.1**</u>, <u>τ_{min} **< 0.4**</u>, <u>**$L/2F$ > 0.5**</u>, <u>η_{max} **> 3**</u> and <u>**$r_{max}/(2F\theta_c)$ < 0.04**</u>) will result to the power transmission coefficient value of <u>**at least 65%**</u> of the maximum possible (in paraboloid case).

Table of concentrator optimal parameters (fig. 11) for fixed source-sample distance 2F = 400 mm and different wavelength

λ, Å	$2r_{max}$, мм	e	L, мм	L_1, мм	L_2, мм	D_{in}, мм	D_{out}, мм	ν	ν/ν_{par}
1.54	<0.16	0.999992	200	6.98	193.02	0.20	1.55	2.64E-4	0.75
9.89	<0.92	0.9998	200	6.91	193.09	1.16	8.83	5.35E-3	0.7
44.8	<2.95	0.998	200	6.19	193.81	3.63	28.29	0.045	0.69
135	<7.82	0.982	200	1.38	198.62	8.34	75.96	0.43	0.82

4. References

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